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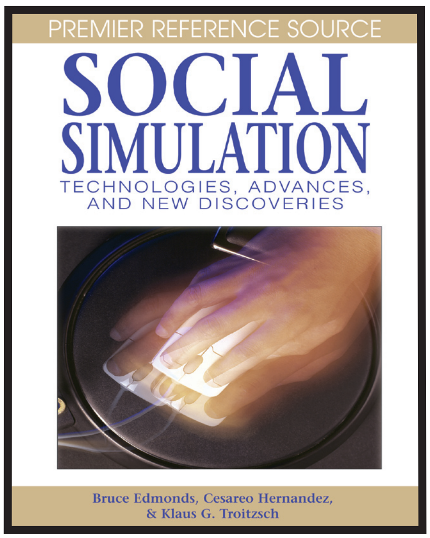

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"In this book some of the models are not primarily meant as a representation at all, but more of a tool to facilitate some interaction between people."
-Bruce Edmonds, Manchester Metropolitan University, UK

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Cooperation as the outcome of a social differentiation process in metamimetic games

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November 5, 2015

Introduction

Where does social differentiation come from? Models of socio-economic systems generally consider agents that pursue some particular ends which are given top-down by the modeler. In classical game theory for example, we often find maximization of material payoffs as the grounding principle of decision-making. Most of the time individual's ends share the following characteristics:

- they are assigned to the agents prior to their social activity and are immutable thereafter,
- in theory these ends could be heterogeneous but in practice they are often all the same i.e. all the agents share the same payoffs' function or these payoffs function evolve under the same selection pressure (which is formally equivalent).

Hidden behind most of socio-economic models we consequently find something equivalent to a social teleology (everybody wants to maximize the same function or the whole system want to minimize a given potential function) which is paradoxically close to the adhesion to a holistic principle. In this line of thought, social differentiation is thus equivalent to a differentiation of means serving more or less the same ends.

This bias is understandable since game theory was originally a normative discipline that would state how people should behave if they wish to achieve certain ends [Luce and Raiffa(1957)]. However, when it comes to understand or reconstruct stylized facts relatively to socio-economic dynamics, this bias is questionable since it is hard to find an end that everybody should pursue and there is no reason to think that particular ends exist relatively to which all the others could be considered as means. Think for example to a person who buys a car. Does she worked to able to buy this car or does she buys this car to be able to go to work ? Indeed, it could be none of these reasons. This crucial point was already raised by Hayek [Hayek(1978)] about economic modeling:

"I now find somewhat misleading the definition of the science of economy as 'the study of the disposal of scarce means towards the realization of given ends' (...) the reason is that the ends which a Catallaxy [1] *serves are not given in their totality to anyone, that is, are not known either to any individual participant in the process or to the scientist studying it."*

This remark changes radically the perspective of social system modeling. If ends are multiple and cannot be listed, what can say the modeler about social dynamics ? How can we speak about social dynamics if the distribution of ends that drives them cannot be identified clearly? This issue was addressed more than a century ago by an eminent sociologist, Gabriel Tarde [Tarde(1898)] who suggested that rather than assuming the existence of an end compared to which all the other are means, we should assume that there is an infinity of possible ends trying to take advantage of one another. This "ecology of ends" and the parameters that influence its dynamics should be studied with the same investment than the well known problem of end-means derivation.

Recognizing this second issue yields an epistemological shift in social systems modeling. From this perspective, the guidelines for the understanding of social dynamics are the different modes of internal consistency of a given distribution of ends in a population. Ends are no more immutable traits that could be assigned to the agents prior to their social activity, their are evolving all along the agents' lives as they evolve in their social environment. This entails the production of diversity with emergence of social groups which is the opposite from the optimization of adaptation. External constraints (like economical constraints or biological ones) appear to act in a second step, as border conditions on the extent of the diversity of possible ends. They determine in no way the dynamics *per se*. This view, in line with Varela's notion of *operational closure* [Varela(1989)], suggests that social differentiation is before all the self-organization of the diversity of possible ends rather than the equilibrium between a multiplicity of means serving the same end. This article will provide an example of such a differentiation process.

But the modalities of this kind of self-organization raises some tricky issues, among others because it requires some kind of self-reference in the definition of the dynamics. We explored this perspective in previous work with the *metamimetic games* framework [Chavalarias(2004)]. This formal framework builds on the fact that human meta-cognition and reflexivity can be introduced as features in formal models so that imitation rules have the property of being their own meta-rules. We can then build on this property to propose models based on a mimetic principle where the distribution of agents' ends in a population is endogenous. The resulting social dynamics is self-referential in the sense that driven by agents' ends it determines the distribution of ends. This distri-

[1] Hayek opposes the notions of *Cosmos* which is a spontaneous order with no purpose and Taxis which is an order that relies on prior ends. *Catallaxy* is the kind of economic arrangement within a *Cosmos* whereas *Economy* is the kind of economic arrangement within a *Taxis*.

bution becomes the outcome of a social differentiation in a cultural co-evolution process.

In this paper, we illustrate this kind of differentiation with a spatial prisoner's dilemma as case study. The aim is to illustrate the epistemological shift such an approach can propose for social systems modeling. Passing, we will see how this approach renew the perspectives on the well known paradox of large scale cooperation in human societies. Before presenting this case study, we will first briefly remind the main features of metamimetic games. A more detailed description can be found in [Chavalarias(2006)].

1 Metamimetic Games

Human beings build their identity learning on their own and through social learning. One of the most important aspects of social learning, and apparently the first in ontogeny, is learning by imitation. This learning skill is exceptionally developed in human beings compared to what exist in other animals species. Human imitation takes several forms, from automatic imitation that is present from birth and seems to persist thereafter in some kind of conformism, to rational imitation where *pro* and *cons* are evaluated before ones engages in an imitation act (what Tarde [Tarde(1890)] called *logical imitation*).

Metamimetic games address this last form of imitation although it might be the case that some kinds of implicit imitation also rely on similar mechanisms. These games are only intended to account for some of the mimetic process of decision-making. They are bound to be coupled with other frameworks for decision-making processes modeling although presenting them separately helps to understand their specific contribution: providing a way to think endogenous dynamics on a set of possible ends in a population of agents.

To understand how ends are represented in the metamimetic framework let us remind a general definition of imitation rule:

Definition: Imitation rule. Given an agent A and its neighborhood Γ_A, an imitation rule is a process that:

1. Assigns a value $\nu(B, \Gamma_A) \in \Re$ to the situation of each agent B in Γ_A. ν is called a *valuation function.*
2. Selects some traits to be copied from the agent(s) in the best situation (according to the target values) and defines the copying process.

For example, in the classical payoff-biased imitation, the value assigned to each neighbor's situation is its payoffs. The agent has then to infer which of the traits of the most successful neighbor(s) are responsible for this success and try to copy these traits. The valuation function here plays a role analogous to the utility function in game theory but here to the difference that it is subjective and evolving through an ongoing social dynamics. It will stand here for the expression of the ends of the agents (who wish to realize a given situation relatively to their relations to their social environment). Two agents can have different valuation functions and the diversity of valuation functions in a population expresses the diversity of ends[2].

To synthesize the above definition, we can say that in step 1, potential models are selected whereas step 2 determines which of these potential models are going to actually influence the agent's behavior and how.

From this definition, the general sketch for metamimetic games is the following. Agents are defined by the actions they undertake in the world plus the rules of decision-making that where used to select these actions (in our case, imitation rules). These rules of decision-making can be organized hierarchically in levels and meta-levels according to which one serves the ends of the other, which one is able to modify the other. These hierarchies of rules defines what we called *metamimetic chains* which are the equivalent of the strategies of the agents in game theory. For example on a financial market, an agent might have for first aim to maximize its profit. To achieve this end she might decide to be temporarily conformist (try to buy what the majority buys) because it is the rule for decision-making that proved to be the most efficient in the current environment. This hierarchy of strategies evolves as the agents update the different levels according to the rules of the above levels (it might happen that since the environment changed, it is more profitable to be in the minority of buyers and consequently a payoffs-maximizer agent will start to play a minority games with other agents).

The three assumptions that define a metamimetic game are the following :

1. **Bounded rationality:** the number k of meta-levels in metamimetic chains is finite and bounded for each agent by its cognitive bound $c_B, (k < c_B)$.

2. **Meta-cognition:** at all levels in a metamimetic chain, imitation rules are modifiable traits. They can be changed for other rules if this is judged relevant by the application of the rule(s) of the above level.

3. **Reflexivity:** imitation rules can update reflexively changing the length of the metamimetic chain in the limit of the cognitive bound of the agents. When the cognitive bound is reached, imitation rules might update themselves.

To give an example illustrating these three assumptions, consider a payoffs-maximizer agent that has only two opportunities of action, C and D. If after reflection she concludes that a conformist behavior is the more successful in

[2]Several terms are used in the literature to define the principles grounding the agents' decisions and actions : ends, goals, aims, motivations, preferences, utility function, values. Although there is undoubtedly a distinction between these terms, at the level of details considered our simple example, they are subsumed under the generic notion of ends. A more accurate model would require to introduce several time scales. For example; Hayek [Hayek(1978)] distinguishes ends and values on their proper time scale. A first study in that direction can be found in [Chavalarias(2004)] where these time scales are properties of the aims considered and depend endogenously on the levels at which their appear in the hierarchical organization of ends

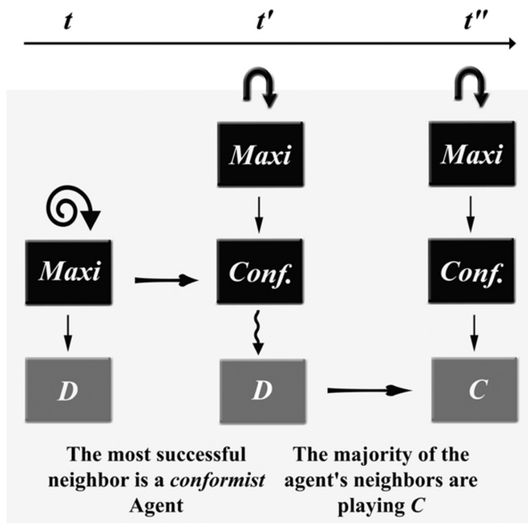

Figure 1: **Endogenous variation in the length of metamimetic chains.** At time t, a maxi-agent A has a conformist neighbor that is more successful than all agents in Γ_A. if A infers that this success is due to the conformist rule, she might adopt this rule as first level rule, and keep in mind that it is only a mean for maximizing her payoffs (meta-level). Thereafter, it might be that according to this conformist rule, the current behavior is not the best one and has to be changed.

terms of material payoffs, she might decide to change her strategy which will change the length of its metamimetic chain as described in figure 1.

But if the cognitive bound of the agent does not allow her to keep in mind the two distinct ends ($C_B = 1$), she might then revise her strategy an drop her main end, as described in figure 2.

It is not the place here to discuss about the relevance of these kind of transition see [Chavalarias(2006)] for more details. Let us just mention three kinds of relations between goals and sub-goals that can be schematized by these transitions:

- The agent adopts a new end and progressively forgets for any reason its old end,
- The new end is so time-taking that although the old end is still present in mind, it is never taken into account in subsequent decisions,
- The agent enjoys more the activities associated to the new end than those associated to the old one and decides to adopt the new one as her main ends.

In all cases, the salient feature is that new ends are adopted because they are

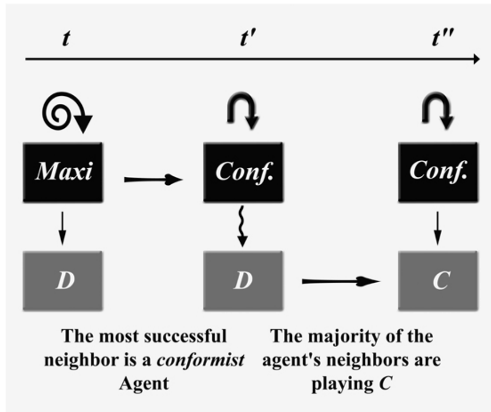

Figure 2: **Reflexive update at the limit of the cognitive bound.** At time t, a maxi-agent A has a conformist neighbor that is strictly more successful than all other neighbors. if A infers that this success is due to the conformist rule, she might adopt this rule at its first level. Since $C_B = 1$ this simply replace the original maxi-rule. Thereafter, it might be that according to this new rule, the current behavior is not the best one, and has to be changed.

consistent with old ones at the moment of their adoption. At a given moment, the set of current ends constrains the way this set can be modified.

Now, if we consider a population of artificial metamimetic agents, with a given set of possible ends, the principles outlined above define the internal dynamics of the artificial social system (described mathematically in previous works by the Markov chain P^0). This dynamics has some stable states, *metamimetic equilibria* that are *counterfactually stable states i.e.* states such that *no agent can find itself better when it imagines itself in the place of one of its neighbors.* More frequently, we encounter stable sets of states, *metamimetic attractors.*

Since agents do errors at the different levels of decision-making process (inference, reasoning, action implementation) and because their environment is noisy, the right object to study *in fine* is a perturbated Markov process P^0_ϵ in the framework of stochastic evolutionary game theory [Foster and Young(1990)]. However in this paper, we will mainly focus on the internal dynamics (P^0).

After these preliminaries, we are now able to give an example of a social differentiation process by cultural co-evolution within a set of multiple ends.

Table 1: The matrix of the prisoner's dilemma game

Player A	Player B	
	C	D
C	(R, R)	(S, T)
D	(T, S)	(P, P)

2 The Spatial metamimetic prisoner's dilemma Game

2.1 The model

Following Nowak et May [Nowak and May(1992)], we will consider a spatial model of evolution of cooperation. We choose this particular model as first example for three reasons. First, the PD dilemma and evolution of cooperation is a scientific puzzle in the consequentialist view of human decision-making. Second, rules for decision-making in the original model of Nowak and May can be interpreted as payoffs-biased mimetic rules which simplify the comparison with the model presented here. Third, the properties and limits of this original model are well known and can be summarized as follow (see for example [Hauert(2001)]):

1. Cooperation is possible in some areas of the parameters' space were there is a coexistence of zones of cooperation and defection evolving constantly with time,
2. These areas of the parameters space are very tiny and correspond to weak social dilemmas. Consequently, cooperation is not sustainable most of the time.

The model can be described as follow. Agents are displayed at the nodes of a two dimensional toric grid. Agents play a prisoner's dilemma game (PD game) each round with each of their neighbors, choosing between two simple actions: cooperate (C) or defect (D). The actions used in the games with different neighbors are the same for a given period. Neighborhoods of players are composed by the eight adjacent cells. When two agents play together, they receive a payoff of R if both cooperate (C) and P if both defect (D). In case their strategies are different, the one who played D receives a payoff of T and the other receives S (*cf.* Table 1). The two conditions for this game to be a prisoner's dilemma are:

1. $T > R > P > S$: defection is always more advantageous from the individual point of view.
2. $T + S < 2.R$: mutual cooperation is the best you can do collectively.

At the end of each period, the sum of the payoffs of each agent are computed and agents update their strategies on the basis of the available information on the last period.

2.2 The set of strategies

We will consider agents with $C_B = 1$. Their strategy will thus be described by a behavior and an imitation rule for decision-making : $s = (b, r)$. Although $C_B = 1$ is not a realistic assumption[3] this will be sufficient to illustrate our purpose. For simplicity, we will also assume that the second step in an imitation process is just pure copying of the trait. The set of rules for imitation should represent all the possible ends that agents can imagine from what they perceive. Consequently, it's important that this set is generated from what the agents can perceive and what kind of processing the agents can do on these perceptions[4]. For this reason, we define the set of imitation rules as outcome of a the result of the combinations of different kinds of cognitive operators: operators for the selection of a particular dimension in the perception space and operators for computation on this selection. we thus have the following scheme:

$$PERCEPTION \rightarrow PROCESSING \rightarrow IMITATION\ RULES$$

Here, we will assume that:

As for the perception, agents can perceive:

- material-payoffs of their neighbors,
- the last action (C or D) of their neighbors,
- the rule they used to choose them.

Here, perceptions are 'exact' and agents are somehow mind-readers. The issue of errors in perception and inference is addressed in [Chavalarias(2004)] but is out of the scope of this article.

As for computation, agents can:

- Compute the proportions of different behaviors and rules in their neighborhood,
- Compare two real numbers and take the max (max of payoffs, max of proportions),
- Multiply by -1 a real number (if agents can imagine an ordering of situations, they can also imagine the inverse ordering, or to say it differently, if they can compute the *max* of two numbers, they can also compute the *min*).

Moreover, we will assume that the agents have zero memory: they can only build rule of conduct that take into account the perceptions relative to last period[5]. This generates four valuation functions and consequently four imitation rules (*cf.* Table 2):

[3] Neurobiological studies seems to indicate that for the 'now and here' we have something like $C_B = 2$ (Etienne Koeklin, personal communication).

[4] Note that in social systems, this is already a cultural construction.

[5] See [Chavalarias(2004)] for a study of the influence of memory

Table 2: The four imitation rules

Computation	Perceived dimension	
	Densities	Payoffs
max	$Conformist$	$Maxi$
min	*Anti-conformists*	$Mini$

1. ***Maxi***: "copy the most successful agent in your neighborhood in terms of material payoffs".
2. ***Mini*** : "copy the less successful agent in your neighborhood".
3. ***Conformism*** : "copy the trait (behavior or rule) used by the majority of agents"
4. ***Anti-conformism*** : "copy the trait (behavior or rule) used by the minority of agents"

In the computational study presented here, each period proceeds with parallel updating as follow[6] (see the detailed algorithm in appendix):

1. Each agent looks at the last period's situation of its neighbors (payoffs, rules, behavior),
2. For any agent A, if according to A's valuation function there are some agents in Γ_A in a better situation than A and if all these neighbors have a valuation function different from A's, then A imitates the rule of an agent taken at random among its most successful neighbors,
3. if according to its (eventually new) valuation function, A is not among the more successful agents in Γ_A, then A chooses at random one of its neighbors with the better situation from last period and copies its behavior (C or D).
4. for each agent, the scores of the eight PD games with its neighbors are computed and the sum is the new material payoffs of the agent.

Note that an agent that does not have a neighbor with a better situation will be satisfied with its own and will not engage in an imitation process. It will just stick to its former strategy.

3 Self-organization of rules and stability of cooperation

We will now give some computational results where we will be able to see this particular kind of differentiation process we mentioned in the introduction. We

[6]In the following we say 'it' for agents since now we are dealing with artificial agents.

will study the influence on this differentiation process of the strength of the social dilemma (parameter p) and of the initial disposition of the population toward cooperation (proportion of cooperators at the first period). All the simulations have for initial state a uniform distribution of the four imitation rules.

These results are qualitatively unchanged in case of asynchronous updating, even with endogenous time constants [Chavalarias(2004)].

We will adopt the following exposition plan:

1. A detailed study for the 'historic' settings of the PD game [Axelrod(1984)]: $T = 5$, $R = 3$, $P = 1$ and $S = 0$, and an initial rate of cooperation of 30%.

2. An extensive study of the influence of the strength of the social dilemma and of the initial disposition of the population toward cooperation : stability of the qualitative properties of the attractors found in 1.

3.1 Emergence of social group and cooperation

Let's begin with a study for a particular set of parameters: $T = 5$, $R = 3$, $P = 1$ and $S = 0$, and an initial rate of cooperation of 30% (figure 3). We report here a study on 50 independent multi-agents simulations with a population of 10 000 agents each. The spatial distribution of the different kinds of rules and behaviors in the population was uniform according to the initial proportions.

The first noticeable facts are that in all simulations, the system quickly reaches an *heterogeneous* attractor while the rate of cooperation *increases*[7]. This attractor is mostly static (only a few oscillators remaining). This means that at the attractor, most agents are counterfactually stable even if almost all possible ends are represented in the population. The attractor is heterogeneous at both behaviors' and rules' levels. The emerging patterns make sense relatively to the ends of the immerged agents and an external observer could even guess who is who in this global picture : conformists are forming large areas where they are in majority, anti-conformists are scattered on all the territory and are locally in minority, maxi and mini agents have interlaced populations, the formers 'exploiting' the others which enjoy.

The interpretation of these emerging structure is that the structure of the attractor reflects the constraints imposed by the self-consistency of the rules, they are is the projections at the collective level of the *elementary virtualities* contained in each agent.

At the behavioral level, we found a mixed population with plain clusters of cooperators and defectors, interlaced areas and scattered exceptions. These structures can be explained only if we look above at the rule level. For example, mini agents exclusively cooperate because it is actually the best way to minimize ones payoffs. It should be emphasized that most of the agents changes both their behavior and imitation rule during their lives, sometimes several times. The original assignment of rules at first period has only weak consequences on

[7] Should we remind the reader that in the original model of Nowak and May, the rate of cooperation would have collapse down to zero with such parameters?

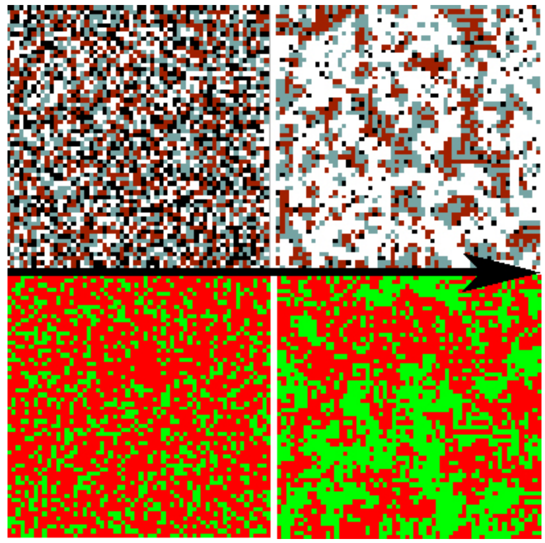

Figure 3: Each small square represents an agent. This configuration is globally stable (only few oscillators remaining at the attractor). *Legend:* Upper part - white: conformists, black: anti-conformists, light grey: mini, dark grey: maxi. Lower part - light grey: cooperators, dark grey: defectors.

what agents will be in their social life. An agent stops changing its strategy when it finally finds a rule and a behavior such that the behavior is consistent with its social environment relatively to its rule. It is this multitude of agents looking for their identity that collectively produce a global stable order. This process is what we call *social cognition*.

Now if we run different simulations with same initial settings and study the path toward the attractors, we can see that (figure 4):

1. All populations reach very quickly their attractors,

2. these attractors are statistically and qualitatively similar as well as the trajectories to reach them (the variance on the distribution of distributions is quite small),

This suggests that the kind of structure an artificial society reaches well constrained by the internal cognitive structure of the agents (the perception and computation operators), the statistical distribution of rules and behavior and the PD matrix. We will now run a sensibility analisis to study the dependence of this structure toward these last parameters.

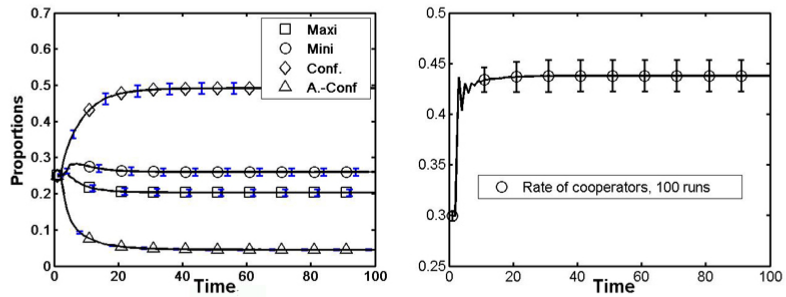

Figure 4: *Left:* evolution of the distribution of imitation rules. Here $T = 5$, $R = 3$, $P = 1$ and $S = 0$, and the statistics have been computed on 50 runs, 10 000 agents each. The population stabilized with about 48% of of conformists, about 27% maxi and 20% mini , and 5% of anti-conformists. *Right:* Statistics on the evolution of cooperation. The rate of cooperators increases from 30% to a proportion of 44% of cooperators.

Table 3: A parametrization of the PD game matrix

Player A	Player B	
	C	D
C	$(1-p, 1-p)$	$(0, 1)$
D	$(1, 0)$	(p, p)

3.2 The influence of environmental constraints

To study the influence of the initial rate of cooperators and the strength of social dilemma on the dynamics, it is more convenient to consider a matrix of the game described by only one parameter. It is well known that two parameters suffice to describe the whole set of distinct games. The problem now is to select a subset of this 2D space that would nevertheless generate all the interesting dynamics. For this purpose, we will take a parametrization frequently used in the social dilemma literature: the payoffs matrix given by table 3. Here p measures the strength of the dilemma: the higher p, the stronger the dilemma.

We will assume that $0 < p < 0.5$ so that the condition $T > R > P > R$ is satisfied. The condition $T + S < 2R$ is violated (we have equality) but it doesn't have noticeable consequences on the dynamics[8].

We did a study similar to the case presented in previous section, for the initial rate of cooperation ($IniCoop$) varying between 5% and 95% and the strength p of the social dilemma varying between 0.1 and 0.4. The same qualitative properties were observed concerning the attractors (fig. 5).

Behavioral level : The rate of cooperation at the attractor is plotted on Fig-

[8] Actually, this condition is often neglected in models

ure 5. We can see that this rate is always above 9.5% and above 40% in the majority of the cases. Attractors at the behavioral level depend heavily on $IniCoop$ for low p but are almost independent of $IniCoop$ for $p > 0.2$. On the contrary, p has always a great influence on these attractors. Even if it is not the point here, it is noteworthy that the high level of cooperation for most of the parameters range is a very interesting result in the perspective of the emergence of cooperation.

Meta-rules level : Here $Inicoop$ has even less influence on the meta-rules than it has on behaviors. The proportions of conformist agents decrease when p increases, while the opposite phenomenon happens with maxi and mini agents. If conformist agents are always the population with the highest density, there is a significant proportion of $maxi$ and $mini$ agents for $p > 0.2$ (more than 20% for each population). On the contrary, the proportion of non-conformists is not sensitive to p and is almost constant along both axes (it seems to be a function of the topology of the social network). At the attractor, most agents perform repetitive behavior without changing anything at their behavioral level or meta-level. However, few agents, at the border of clusters, keep changing one of these two traits. We will see why in next section.

3.3 Why can't they fix their mind ?

To understand why some agents can't be counterfactually stable and keep on changing their mind at the attractor, let's study the case of a particular agent, *Eidaid* that keeps oscillating between conformism and payoffs' maximization. This agent is at the border between a conformist area and a maxi area (cf fig. 6). All her neighbors are defectors, itself included.

It is easy to see why *Eidaid* can't keep the conformist rule : the majority of *Eidaid* neighbors are maxi-agents. Then, each time *Eidaid* becomes conformist, it will update its rule to maxi the next period. On the other hand, a study of payoffs' distribution reveals that the most successful agent in *Eidaid*'s neighborhood is *Eidaim*, a conformist agent. The reason is that *Eidaim* has the chance to have an anti-conformist neighbor, which is playing C with it. Thus, each time *Eidaid* adopts the maxi rule, it will copy *Eidaim* next period, and become conformist again. Endlessly.

We can see here that the topology of the social network is crucial for this kind of phenomena. It is precisely because *Eidaid* wants to imitate some neighbors without having the same neighborhood that it always hesitates between the two rules. The fact that this kind of configurations can only happen at the border between two social groups (defined by homogenous behaviors or meta-rules and behavior) combined with the fact that the whole population is strongly structured explains why there are so few oscillators.

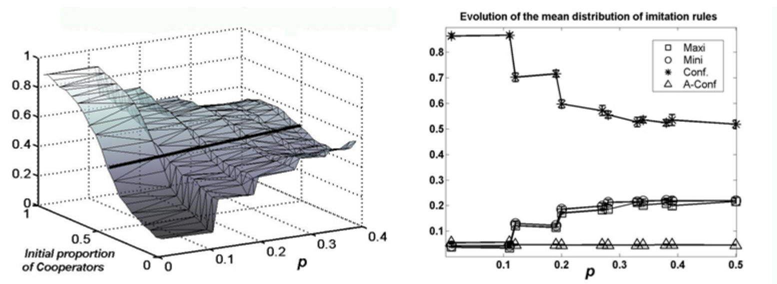

Figure 5: **Influence of initial propensity for cooperation and the strength of the social dilemma on the structure of the attractors:** *Left:* Dependence of the rate of cooperators at the attractor (100 time steps) in function of parameters p and the initial rate of cooperators $IniCoop$. Each point represents the mean rate of cooperation (on 10 independent runs) at the attractor for the couple $(p, IniCoop)$ considered. The rate of cooperation is always above 9.5%. Cooperation is sustainable under all the conditions studied. The line represents the set of simulations corresponding to the area of parameters used in the right figure. *Right:* Evolution of the distribution of metamimetic rules in function of p, for an initial rate of cooperation of 50%. Each plot represents the mean evolution of the proportions at the attractor of the corresponding imitation rule in function of p (on 10 independent runs). We can see that conformists (stars) are predominant under all condition but maxi (sqares) and mini (circles) agents are more represented in areas corresponding to high p than in those corresponding to low p. Error bars represent the standard deviation. We have : $0.01 < p \leq .5$ and $1\% < IniCoop < 99\%$.

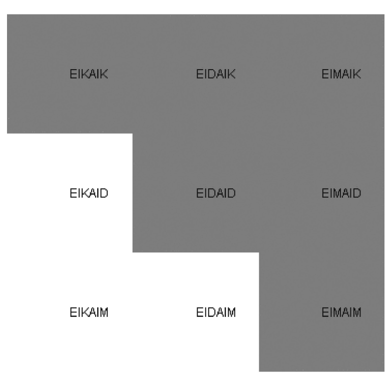

Figure 6: Neighborhood of agent *Eidaid*. White agents are conformist, grey agents are maxi.

3.4 Counterfactual stability and self-organization of ends

The right graph of figure 5 is interesting because it predicts some qualitative trends for behaviors in the spatial prisoner's dilemma that could be verified experimentally. Given the simplicity of the model, there are much chance that such verifications would turn to refutations but it is interesting to analyze more in details this graph because this will gives some ideas about the kind of issues that can be addressed with such metamimetic models. The main predictions are the followings:

Influence of the initial rate of cooperation: The initial rate of cooperation has a positive influence on the rate of cooperation at the attractor. For a given p, this dependence has a S shape along $IniCoop$, which is the well-known signature social learning. We can thus expect that this shape is due to some kind of conformist behavior.

Influence of the strength of the social dilemma: As we can see, the surface of the graph becomes almost flat as p increases. This means that the higher the strength of the social dilemma, the lower the influence of the initial propensity of the population for cooperation. This leads to very counterintuitive results. For example, for a initially low proportions cooperators, the proportion of cooperators at the attractor increases when the strength of the social dilemma increases. From 1, we can also expect that the weight of conformist agents in the population will decrease as p increases if we study the dependance at the rules' level.

To explain these qualitative results, we actually have to look at the rules' level (left graph of figure 5). We can observe that the distribution of rules for imitation varies as p increases from a almost all-conformists society to a mixed population of conformists, maxi and mini agents. When p is low, the dynamics of the game is thus close to a game between conformist agents: initial conditions matter and the higher the initial rate of cooperation, the higher the final rate of cooperation. When p is high, populations of maxi and mini are sufficiently important to influence the behavior of conformist agents. Since these two population are in close proportion and since they tend to adopt opposite behaviors (C for mini and D for maxi), these two behaviors will be almost equally distributed in the conformist population as well with a slight advantage to C behavior (maxi are more represented that mini). This account for the high rate of cooperation in the whole population.

This explains roughly why the structure of the behavioral level at the attractor is what it is. In models where ends are given top-down, turning these intuitions into analytical results would be the end of the story, but here, the stake is quite different. We have to explain why the rules' level behaves like this. The distribution of rules at the attractor depends endogenously on the initial values of p and $IniCoop$. Can we explicit qualitatively this dependence? This explanation requires that we enter in the internal logics of the diverse rules and try to track their dependencies. Because of the high complexity of these systems, we will not provide exact analytic results but only try to give qualitative intuitions.

$Conformists$ and *anti-conformist* don't care about material payoffs *per se*. Consequently, they should not be affected by variations of p in their decisions about new rule adoption opportunities. It is the same for variation in $IniCoop$ since the decision about adopting a new rule is only indexed on the proportion of this rule in the neighborhoods. On the contrary, the behavior of $maxi$ and $mini$ depends heavily on payoffs. But, it is not the absolute value of payoffs that affect the behavior through variations of p but the way p influences on the disparities of payoffs. Indeed, a linear transformation of payoffs does not affect the decision of these type of agents.

To understand this, we have to enter into the logic of these types of agents. For example, what is it to be in the skin of a maxi agent? At the beginning of the game, you have to find the strategy that will give you the best payoffs in your neighborhood. But the environment is changing quickly in these first periods of the game, every neighbor is looking for an identity, imitating those they think to be the bests. In particular, the configurations of behaviors and consequently of payoffs are fluctuating. As maxi, there is little chance you play C for long time if you begun to do so because you will soon have a neighbor playing D that will beat you. So unless you drop your rule for an other one, you will quickly adopt the D behavior. But still, there are some cases where you can be supplanted by one of your neighbor more lucky with its own neighbors (more neighbors playing C with it). If this neighbor is not a maxi-agent, you will be tempted to copy its rule. The counterfactual stability of maxi-agents will thus be positively correlated with the probability for an agent playing D to be disappointed when it compares its material payoffs with one of its neighbors. We can give an estimation of this probability in function of p and the initial propensity of the population for cooperation when behaviors are uniformly spatially distributed.

- Given p and $IniCoop$, the probability for a given agent A to have k neighbors playing C is :$w(k) = C_8^k.IniCoop^k.(1 - Inicoop)^{(8-k)}$;
- The corresponding payoffs are $g_C(k) = k.(1 - p)$ if A is playing C and $g_D(k) = k + (8 - k).p$ A is playing D.
- The distribution of payoffs in the population has thus for modes $g_D(k)$ and $g_C(k)$ with weights $(1-IniCoop).w(k)$ and $IniCoop.w(k)$ respectively. Let F be the cumulative distribution of these payoffs.

To have a more precise idea of what happens, we can plot the distribution of payoffs relatively to the behavior of the agents (fig. 7).

We see that there are two issues. The first is that in some spatial configurations, an agent playing D can have lower payoffs than one of its neighbors playing C. This is what we call the degree of *spatial dominance* of D on C in the space of spatial configuration. The second is that an agent playing D can still have a neighbor playing D with higher payoffs. This is linked to the

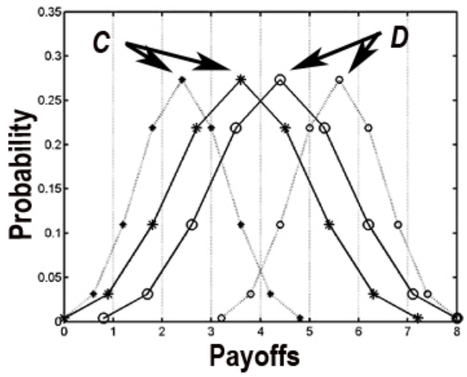

Figure 7: Plot of the probability distribution of payoffs in function of the behavior of the agent (C: -*- ; D: -o-) with a disordered initial state comporting 50% of cooperators, for $p = 0.1$ (black line) and $p = 0.4$ (gray line). The degree of overlapping between two plots for a given p indicates the degree of *dominance* of D on C; the variance of these distributions indicates the *residual uncertainty* when a payoff-biased agent has adopted the dominant strategy.

variance of the payoffs' distribution for each action C and D and will have for consequences that a maxi-agent (and symmetrically a mini-agent) will have a higher probability to question the fitness of its rules when this variance is high. An increase in p both reduces this variance and increases the degree of dominance of D on C. Consequently, we might expect that the higher p, the more counterfactually stable are mini and maxi agents.

We can see it more precisely. After what is above, the probability for a defecting maxi-agent to be disappointed when it compares itself to one of its neighbors can be approximated with :

$$I_{maxi} = \sum_{k=0\ldots8} w(k).(1 - F(g_D(k))$$

From the plot of this counterfactual instability index we can predict two main qualitative traits concerning the dependence of the maxi rule distribution on p and *IniCoop* (fig. 8). First, this index decreases with some discontinuities[9] when p increases: the higher the strength of the social dilemma, the higher the counterfactual stability of maxi-agents. Second, this index has an inverted U-shape along the *IniCoop* axis and the U gets distorted as the strength of the social dilemma increases: maxi-agents are relatively more counterfactually stable when the initial propensity of the population for cooperation is high than when it is low.

[9]The discontinuities are well-known consequences of the fact that payoffs are discrete.

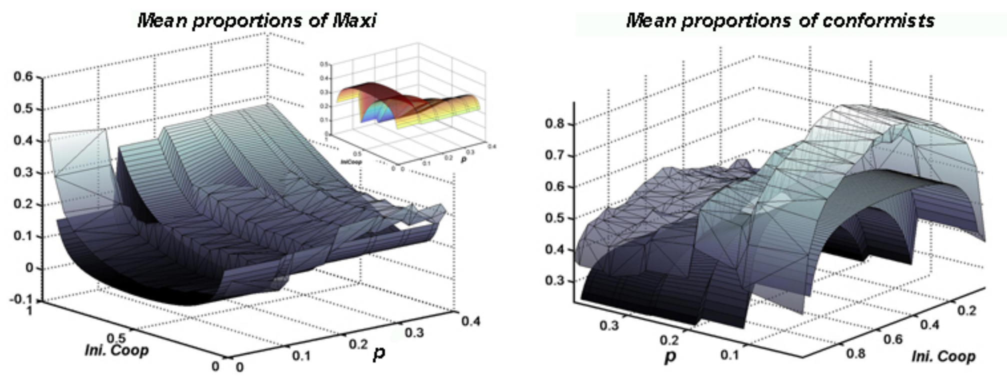

Figure 8: Evolutions of the distribution of rules in function of the strength of the social dilemma (p) and the initial propensity of the population for cooperation (*IniCoop*). *Left:* On the same plot the mean proportions of maxi-agents at the attractor and the index of counterfactual instability I_{maxi}. The index is inversely proportional to the mean proportions of maxi. It has thus been multiplied by -1 and translated to ease the comparison with simulations' results. The small figure in the upper right corner is the index without any transformation. *Left:* The evolution of conformism in function of the strength of the social dilemma and the initial propensity of the population for cooperation plotted against the index $1-(1-I_{maxi}).(1-I_{mini})$. We can see that variations of this index are a well correlated with the variations of proportions of conformists agents.

These predictions are corroborated by the multi-agents simulations (fig.8). Moreover, we can deduce from the above the evolution of conformism in the population in function of the strength of the social dilemma (p) and the initial propensity of the population for cooperation. The conformist group will benefit from the counterfactual instability of maxi and mini. Consequently their proportions should be positively correlated with the quantity $1-(1-I_{maxi}).(1-I_{mini})$. This is exactly what we can observe when we compare the proportion of conformists with this index.

With this qualitative insight, we can better understand the evolution of the rate of cooperation in function of the strength of the social dilemma and the initial propensity of the population for cooperation. The shape of the surface plotted in figure 5 is now more understandable and as we saw it, the explanation lays inside the internal consistency of the set of rules at the attractor. More over, this insight concerns all the parameter space for this given initial uniform distribution of imitation rules. From this study, the two critical factors are:

- The degree of *spatial dominance* of a behavior (here D on C) (*cf.* fig.7): Since in metamimetic games we deal with imitation and counterfactual situation, the spatial dominance takes into account spatial configurations involving neighbors and second neighbors, contrary to game theory where dominance is defined in terms of outcomes with direct partners.

- The *residual uncertainty*: this is related to the variance of payoffs on spatial configurations of neighbors and second neighbors for a given behavior. Even when there are some behaviors that spatially strictly dominate the others, two players with the same behavior can have different payoffs because they have different opportunities with their own neighbors. This could lead the counterfactual instability of an agent.

4 Conclusion: optimization vs. internal dynamics?

- Well mister A., I am afraid you loose! your are in the minority.

- But that's exactly what I want, I'm anti-conformist!

This short dialog illustrates the impossibility to interpret all socio-economic behaviors in terms of an optimization process, especially if this does not take into account the diversity of ends. In a previous paper [Chavalarias(2006)] we have outlined a formalism for social systems modeling designed to take into account the self-organization of a multiplicity of ends, distinguishing between the internal dynamics of a social systems (P^0) and its coupling with the environment (P^0_ϵ). We gave here an illustration of what internal dynamics look like with a case study on a spatial prisoner's dilemma.

Contrary to most models explaining cooperation in social systems as the output of an optimization process, we have introduced a metamimetic principle and explained cooperation as the outcome of a spontaneous differentiation process under cultural co-evolution. In this framework, the question is not the traditional "How can altruists 'survive' in a selfish worlds?". Rather, in line with [Tarde(1890)], the main issue is to understand how heterogeneous ends can reinforce or limit each others in their own identities to collectively entails the emergence of the observed patterns. From this point of view, cooperation is no more a paradox since, contrary to what can be thought, when imitation acts at the meta-level, the mimetic dynamics leads to heterogeneity of ends and consequently of behaviors. This process of differentiation, *social cognition*, is the signature of social systems which have the particularity to endogenousely change themselves.

Now that the picture of the internal dynamics of social systems is drawn, we have to explore its coupling with an external environment that imposes viability constraints and introduces noise in the internal dynamics. This was sketched in [Chavalarias(2004)], great questions are still ahead.

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Appendix

The algorithm used for the simulations presented in this paper is the following:
Set up of the game:

- Give a value for p, $0 < p < 0.5$.
- Agent at displayed on a toric grid, their neighborhood are composed by the eight adjacent cells.

Initial Conditions:

- Give the spatial distribution of imitation rules. Here, there was four rules. For each agents, we assigned randomly one of the rules with a probability 1/4
- Give the spatial distribution of behaviors. Here, for each agents, we assigned the behavior C with a probability $IniCoop$ and D otherwise.

At each period, for each agent, with parallel update at the population level[10]:

- For each agent, the scores of the eight PD games with its neighbors are computed and the sum is the new material payoffs of the agent.
- Each agent looks at the last period's situation of its neighbors (payoffs, rules, behavior),
- For any agent A, if according to A's valuation function there are some agents in Γ_A in a better situation than A and if all these neighbors have a valuation function different from A's, then A imitates the rule of an agent taken at random among its most successful neighbors,
- if according to its (eventually new) valuation function, A is not among the more successful agents in Γ_A, then A chooses at random one of its neighbors with the better situation from last period and copies its behavior (C or D).

[10] We remind that the parallel update do not generate artifacts [Chavalarias(2004)].

References

[Axelrod(1984)] Axelrod, R., 1984. The Evolution of Cooperation. NY: Basic Books.

[Chavalarias(2004)] Chavalarias, D., 2004. Métadynamiques en Cognition Sociale. Ph.D. thesis, Ecole Polytechnique, Paris, France.

[Chavalarias(2006)] Chavalarias, D., 2006. Metamimetic games: Modeling Metadynamics in Social Cognition. Journal of Artificial Societies and Social Simulations Vol. 9, Issue 2.

[Foster and Young(1990)] Foster, D., Young, P., 1990. Stochastic Evolutionary Game Theory. Theoretical Population Biology 38, 219–232.

[Hauert(2001)] Hauert, C., 2001. Fundamental clusters in spatial 2×2 games. In: Proc. R. Soc. London B. vol. 268, pp. 761–769.

[Hayek(1978)] Hayek, F.A., 1978. New Studies in Philosophy, Politics, Economics and History of Ideas, Chicago Univ. Press, chap. The Confusion of Language in Political Thought. pp. 71–97.

[Luce and Raiffa(1957)] Luce, R.D., Raiffa, H., 1957. Game and Decision: Introduction and critical survey. Paperback.

[Nowak and May(1992)] Nowak, M., May, R.M., 1992. Evolutionary games and spatial chaos. Nature 359, 826–829.

[Tarde(1890)] Tarde, G., 1890. Les lois de l'Imitation. Jean-Marie Tremblay, Les classiques des Sciences Sociales edn.

[Tarde(1898)] Tarde, G., 1898. Les lois sociales. Esquisse d'une sociologie. Jean-Marie Tremblay, Les classiques des Sciences Sociales edn.

[Varela(1989)] Varela, F.J., 1989. Autonomie et connaissance. Seuil.