Modeling endogenous preferences for heterogeneous agents adressing “the hard part”

David Chavalarias*and Erasmo Batta
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1 Introduction

In its Prize Lecture [1], Vernon L. Smith sketched what he considered as one of the most important challenges for the future of game theory in the perspective of the modeling of socio-economic systems:

> “Technically, the issue can be posed as one of asking how most productively to model ”agent types” by extending game theory so that types are an integral part of its predictive content, rather than merely imported as an *ex-post* technical explanation of experimental results. For example, moves can signal types, and effect decision, which explains why game form matters, and why payoffs available, but foregone, can effect outcomes. These elements must be part of the internal structure of the theory such that outcomes become predictions conditional on the elementary characteristics of players who read each other’s intentions. [...] The point that needs emphasis is that it is easy to go from “types” (traditionally utility or beliefs about states) to game theoretic choice ; the hard part is to relate “types” to characteristics of the individual’s memory-sensory system.”

In this paper, we address issues related to this “hard part”, which are also strongly linked to the problem of accounting for the heterogeneity of agents in socio-economic systems. The core questions is: How can we build models which exhibit endogenous heterogeneity in agent types (or endogenous preferences) and how could we link this heterogeneity with environmental or economical constraints ?

The relatively recent interest for agent’s heterogeneity owes much to research in experimental economics, and in particular research about cooperation (prisoner’s dilemma, common pool resources, ultimatum games, trust games) which have continuously reported heterogeneity in agents’ inferred preferences, giving at the same time evidences of the failures of rational choice theory. For this reason, we will illustrate our findings with an application to the modeling of cooperation.

2 Modeling agent types heterogeneity with metamimetic games

The framework of *metamimetic games* [2] is appropriate to adress the “hard part”. As we will see it, types distribution is “integral part of its predictive content” in this class of games, and it is possible to endogenize a rich variety of types attributes.

After a brief presentation of this framework, we will show on a toy model how we can adress important issues of social sciences with new insights: how can we model social dynamics with interpersonally not comparable utilities? How to adress the question of social welfare if heterogeneous types distribution (implying interpersonally not comparable utilities) depends on socio-economic policies[1] ?

In this section, we will synthetically present this framework, taking Smith’s view as guidelines.

“The hard part is to relate “types” to characteristics of the individual’s memory-sensory system.” In most micro-economic models, types are defined as rules of behaviour which given some information collected from the environment, the agent’s past experience and the agent’s utility function, define an action to be taken at every decision step. Utility function is one of the determinant factor in agent’s types.

To sketched out how is ”biology providing abstract function defining potential, and culture shaping the emergent forms that we observe”, we will derive these utility functions from the agents’ cognitive endowment.

let us consider a minimal model which mixes materialistic and non-materialistic individuals (non comparable utilities) playing a game G with two possible moves C or D (which will stand for cooperation of defection in our case study). In all gener-

*Corresponding author. Institut des Systmes Complexes de Paris Ile-de-France & CREA, CNRS - Ecole Polytechnique 57-59 rue Lhomond, 75005, Paris, France ; david.chavalarias at polytechnique.edu, http://chavalarias.com

[1] Most social welfare approaches assume that preferences are fixed and propose to compare social states accordingly. But if preferences distribution *depends* of social states, then the problem is different.

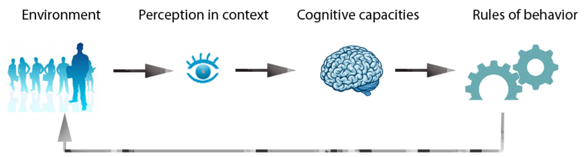

Figure 1: Agents' type are the result of the embedding of agents with particular cognitive endowment into some particular context of socio-economics interactions. Rules of behaviour and agents' types are themselves part of the information which could be inferred from the environment and can enter into play in the definition of rules of behaviour in a more or less sophisticated manner.

ality, agents are embedded in a population, with a set of neighbours with which they play the game G pairwise. Payoffs of the game from players and their partners are the first type of information which can be processed to define utility functions. The second is the observed moves of their partners and their inferred types.

We assume minimal processing capacities on these two types of information. Concerning ordinal values (*e.g.* payoffs), agents are able to compare two figures and take the bigger or the lower. They are consequently able to process the minimal and maximal payoffs in their neighbourhood. Concerning cardinal quantities (*e.g.* the distributions of moves in their neighbourhood), agents are able to process frequencies to convert this information into ordinal quantities. They are thus able to know what is the majority of behaviours for example.

These minimal hypothesis on agents' cognitive endowment generate four possibilities for the utility functions:

- **Payoffs maximization:** utility is higher when payoffs are higher,
- **Payoffs minimization:** utility is higher when payoffs are lower,
- **Conformism:** utility is higher when agent's strategy is similar to a larger number of neighbors,
- **Anti-conformism:** utility is lower when agent's strategy is similar to a larger number of neighbors.

"To go from "types" to game theoretic choice", we will consider simple mimetic agents like for example [4]. Agents measure the performances of their neighbours and their own performance with regards to their utility function ; and copy the strategy of the most successful agent if they are not themselves one of the best. Agent's types will be named after their underlying utility function, with two payoffs based (or materialistic) types, *maxi*, *mini* and two non-materialistic types *conformist* and *non-conformist*.

At this stage, the adoption of a generative model for utility function guaranties the heterogeneity of the population, however, we still have to introduce elements to our framework to make the distribution of types an "integral part of its predictive content". We will adopt the framework of metamimetic games [**?**] which assumes that agent can reflect on their rules of behaviour such that these latter are integral part of the strategy of the agents. Agent are considered as reflexive in the sense that they know the criteria or values upon which they base their choice ; and can take the initiative to change them if necessary. Such statements about strategies are common in literature. For example, according to Smith [5], "it is shown that the investor who chooses to maximize expected profit (discounted total withdrawals) fails in finite time. Moreover, there exist a variety of non-profit-maximizing behaviours that have a positive probability of never failing. In fact it is shown that firms that maximize profits are the least likely to be the market survivors." In the proposed model, reflexivity means that agents have the capacity to change their rule of behaviour if they judge that it is not the best rule to achieve their goal. We thus have a dynamics of types which depends, among other, of their spatial distribution.

There are various options for the procedure of types' change. For example, if conformist agents are judged to be the wealthier in terms of payoffs, a maxi agent could introduce a small proportion of conformism in its strategy ; or simply become conformist. Because we want to capture the essence of the consequences of types endogenization, we will consider the simpler option, which is the latter.

Given all the above, we will consider a metamimetic game where interactions take place as follows for every period of the game:

1. each agent looks at the situation of other agents in its neighbours Γ_A, (payoffs, rules, behaviour),
2. for any agent A, if according to A's utility function there are some agents in Γ_A more successful than A and if all these successful neighbours have a rule of behaviour different from A's, then A copy the rule of an agent taken at random among this set,
3. if according to its (eventually new) rule of behaviour and its associated utility function, A is not among the most successful agents in Γ_A, then A chooses at random one of its neighbours with the better situation and copies its behaviour (C or D),
4. each agent plays the game G with its neighbours using the same behaviour (C or D). Then for each agent, the scores of all its pairwise games are computed and the sum is the new payoffs of the agent.

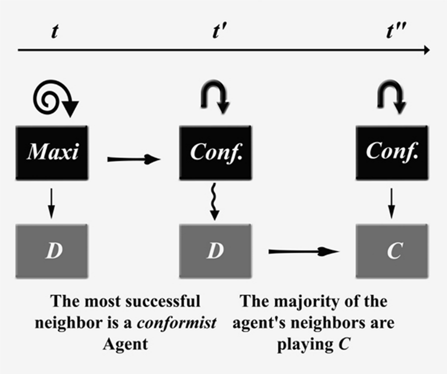

Figure 2: **Rule acting as its own meta-rule.** At time t, an agent A described by the strategy (rule=maxi, behaviour= D) has a conformist neighbour B which is strictly more successful than all other neighbours. If A ascribes the success of B to its conformist rule, it might adopt this rule replacing its original maxi-rule. Thereafter, it might be that according to this new rule, the current behaviour is not the best one, and has to be changed.

This dynamics is illustrated by figure 2. It is important to note that an agent which is the best of its neighbourhood according to its utility function will be satisfied and will not engage in an imitation process at the rule level or at the behavioural level.

The concept of equilibrium associated to this kind of game are *metamimetic equilibria* with are *counterfactually stable states i.e.* states such that *no agent can find itself better when it imagines itself in the place of one of its neighbors.* However, since we are considering evolutionary games with potentially noisy dynamics, we will more frequently encounter stable sets of states, *metamimetic attractors*. In order to characterize these attractors and render them somehow comparable, we introduce a measure of satisfaction of the agents.

We will note $\nu_A(B,\Gamma_A)$ the utility attributed by A for being in the place of B, given the information available in Γ_A. For example, if A is a maxi agent, $\nu_A(B,\Gamma_A)$ will be B's payoffs. If A is conformist, $\nu_A(B,\Gamma_A)$ will be the density of B behaviour or rule (according to what is evaluated) in Γ_A. Consequently, an agent can compare its own situation with the one of a neighbour comparing $\nu_A(A,\Gamma_A)$ and $\nu_A(B,\Gamma_A)$.

We will thus assume that if $\nu_A(A,\Gamma_A) - \nu_A(B,\Gamma_A) < \epsilon$, with $\epsilon > 0$[2], A will consider that A and B are performing equally well. Moreover, instead of considering the absolute value $\nu_A(A,\Gamma_A)$ and $\nu_A(B,\Gamma_A)$ for inter-personal comparison assessment, we will consider that inter-personal comparison is based on normalized value with respect to the agent's neighbourhood, which is in line with results in experimental psychology. We thus consider the *satisfaction* $S(A,\Gamma_A)$ to be equal to 1 when $\nu(B_{max},\Gamma_A) - \nu(B_{min},\Gamma_A) < \epsilon$ and otherwise:

[2] In these work we stablish $\xi = \frac{1}{8}$.

$$S(A,\Gamma_A) = \frac{\nu_A(A,\Gamma_A) - \nu_A(B_{min},\Gamma_A)}{\nu_A(B_{max},\Gamma_A) - \nu_A(B_{min},\Gamma_A)}$$

where $B_{max} = max_{B\in\Gamma_A}\nu_A(B,\Gamma_A)$ and $B_{min} = min_{B\in\Gamma_A}\nu_A(B,\Gamma_A)$.

$S(A,\Gamma_A) < 1$ means that A is not satisfied and will change its strategy (rule or behaviour) at the next decision step. Consequently, if we note P the population of agents and $\Psi = \Sigma_{A\in P} S(A,\Gamma_A)$ we have $\Psi = 1$ at a metamimetic equilibrium and $\Psi < 1$ otherwise. Ψ is the average satisfaction of agents in the population and is an indicator of how much unstable social dynamics are. Ψ will be used to compare different configurations of the game.

3 Spatial prisoner's dilemma with heterogeneous types

To illustrate the insights brought by metamimetic games to the modelling of agent heterogeneity, we will apply this framework to the modelling of prisoner's dilemma. In the following, we will consider a spatial prisoner's dilemma G [4] with the game outcomes parametrized by the parameter p (cf. table 1).

	Cooperate	Defect
Cooperate	$1-p$, $1-p$	0 , 1
Defect	1 , 0	p , p

Table 1: Payoff function of the game played by agents.

p represents the strength of dilemma and for $p \in [0, 0.5]$ the game corresponds to a Prisoner's Dilemma. Agents are displayed at the nodes of a two dimensional toric grid and Γ_A is the Moore neighbourhood (height adjacent cells). The game is initialized with a random uniform distribution of rules and a random distribution of behaviours with an average level of cooperation equals to *IniCoop*. The model has thus two parameters, p and *IniCoop*.

This game has been studied in details in [6] through computational and analytical methods. We summarize here the main results and extend them in the next section:

- Populations reach very quickly an heterogeneous attractor (within dozens of periods). All rules are represented at the attractor (*cf.* figure 3),

- Populations at the attractor are well structured with patterns reflecting the cognitive endowment of the agents (mixed groups of mini and maxi, dense groups of conformists, isolated anti-conformists),

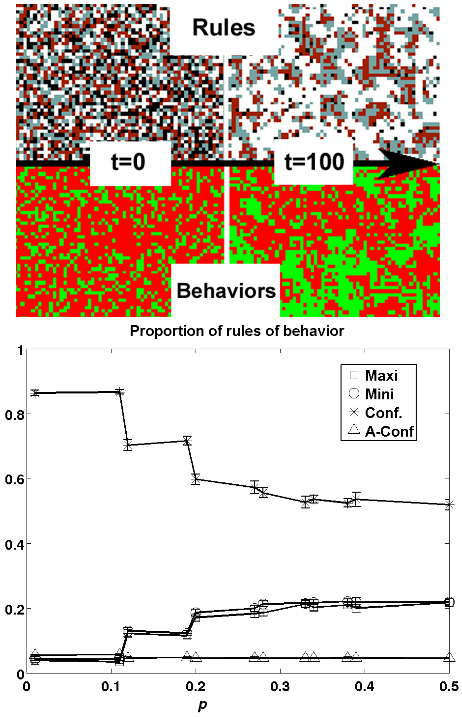

Figure 3: **Emergence of heterogeneous attractors.** *A - right):* Evolution of the spatial distribution of rules (upper part) and behaviours (lower part) from the initial disordered state to a structured attractor. Each small square represents an agent. This configuration is globally stable (only few oscillators remaining at the attractor). *Legend:* Upper part - white: conformists, black: anti-conformists, light grey: mini, dark grey: maxi. Lower part - light grey: cooperators, dark grey: defectors.
B - left): Influence of the strength of the prisoner's dilemma on distribution of rules at attractor for an initial rate of cooperation of 50%. We can observe that environmental factors like the strength of the social dilemma do influence types distribution and that in this example, materialistic types are favoured by strong social dilemma. Discontinuities are due to the discrete character of the neighbourhoods.

- Cooperation is always present at the attractor (between 10% and 90% according to the settings) and the level of cooperation depends both on p and the initial level of cooperation. The influence of the initial level of cooperation decreases when the strength of the social dilemma p increases.

- Variations in types proportions at the attractor are well predicted by the concept of *spatial dominance*[6], which is a quantity approximating for each type, the probability of an agent to be satisfied.

These preliminary results demonstrate that the introduction of reflexivity in mimetic models makes it possible to endogenize agents types such that their distribution becomes "integral part of [their] predictive content". They also give insight on how culture can shape "the emergent forms that we observe" [1] without assuming any other particular selection process. Last but not the least, it supports the idea that cooperation is the product of agents heterogeneity and not some kind of fitness optimization.

4 Noisy dynamics

The above results have been obtained on systems which can in all generality can be described as Markov processes, where the states are the spatial configurations of rules and behaviours of agents. As things stand, these Markov processes are not ergodic, which is reflected among other things by the influence of initial conditions (*e.g.* initial rate of cooperation) on the attractor. However, real socio-economic systems face a large variety of perturbations and uncertainties, such that it is reasonable to assume that they should be modelled as noisy systems. From the modelling perspective, as stressed in [2], this means that we should consider the attractors of the perturbed Markov process, which have all chances to be ergodic. Attractors shall depend on the kind of perturbations or uncertainties applied to the system and it is not the place here to discuss the variety of options. To catch the essence of these perturbed dynamics, we will consider a natural source of noise in the modelling of social systems induced by the necessary limited lifespan of the agents. We decided to represent "one year" by twelve games[3] and used data from the 2010 US Census Bureau [10] to initialize agents age and compute at each period the probability for an agent to die in function of its age, thus defining a replacement dynamics. Every died agent is then replaced by a new agent which rule and behaviour are randomly selected among the set of possible choices.

The introduction of ergodicity in the model has two consequences:

- The initial rate of cooperation has no more influence on the attractor,

- The dependence of the systems toward p is similar to what is shown on figure 3-B, with an increased proportion of payoffs-based types as the strength of the dilemma increases and rate of cooperation oscillating around 50%.

In that case, types distribution does not depend any more on initial conditions, the only parameter being the strength of the social dilemma p. This approach is particularly interesting for modelling situations where p can be interpreted as a political leverage (for example employment legislation or collective agreements could be thought as instruments to modify the employment security, trust, and effort dilemma on the job market [7]). In that case, this approach makes it possible investigate how political decisions could impact on the evolution of

[3] The number of games per year is a neutral variable from the moment it is sufficiently large for the cultural dynamics to really play its role.

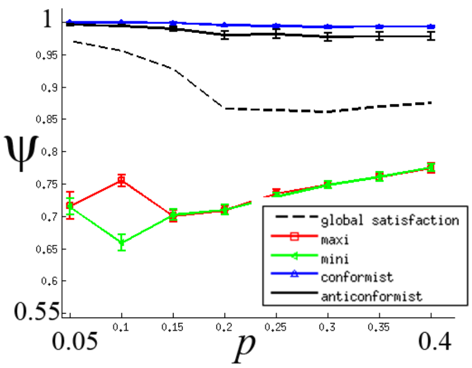

Figure 4: **Effect of the strength of the social dilemma p on the satisfaction of agent at the attractor.** Agents types have different satisfaction levels, payoffs-based types being the less satisfied, which has already been underlined by several psychological studies. The population satisfaction Ψ is significantly higher for low strength dilemma compared to high strength dilemma.

preferences in the population, a phenomena which importance has been stressed by [8]. Moreover, the population satisfaction Ψ gives a natural indicator on the population well-being at the attractor. Even if the spatial prisoner's dilemma studied here is too simple to allow a direct analogy with real-word issues, it is noteworthy, as shown by figure 4 that Ψ is a decreasing function of the strength of the dilemma and that there is an heterogeneity in the satisfaction of agents type, payoffs-based types being the less satisfied. This phenomena has already been underlined by several psychological studies [9].

5 Generic method for the endogenization of agents trait

Agent types are generally described by several kind of traits, their utility function being one of them. Each time a trait is assumed to be part of the agents strategy and could be somehow inferred by other agents, it could be endogenized through metamimetic principles. In the previous section, we saw how to endogenize discrete traits like the rule of behaviour. In this section we will focus on the endogenization of continuous traits with the example of time scales. Time-scales and their hierarchies are an important issue in socio-economic systems modelling although this issue is hardly addressed. For example, it is well known that several models of spatial dilemma are critically sensitive to the synchronous or asynchronous aspect of the behaviours update[4]. To check that this is not the case for our previous results and investigate the structures emerging from time-scale endogenization, we will assume that instead of updating their strategy every time-step, agents do it stochastically. When an agent is not the most successful of its neighbourhood, we assume that it engages in a rule updating process with a probability $\theta_r \in]0,1]$ and in a behaviour updating process with a probability $\theta_a \in]0,1]$.

In this new game θ_r and θ_a are part of agents strategy and consequently can be imitated. We will assume that the inference of continuous traits, like time scales, can be done up to a given precision by adding a noise ξ to the measurement[5]. Moreover, we will assume that agents can update a continuous traits α by weighting it with the traits α_{best} associated to the best rule: $(1-\nu)*\alpha+\nu*(\alpha_{best}+\xi)$. The influence ν appears here as an additional parameter. In principle, it could also be endogenized, which was not the chosen option in the following computational studies. However, we did a sensibility study on ν and it reveals that all results are qualitatively similar for $\nu \in [0.1, 0.9]$. Extreme ν values reveal particular dynamics but are not realistic from the psychological perspective and are moreover associated with pathologically low levels of Ψ.

The particular values for the initialisation of θ_r and θ_a are not important. The only thing which matters is first, the order of magnitude of the ratio between these initial values, which determines the baseline time-scale of the cultural dynamics ; and second, the lifespan expectancy of the agents. For that reason, we choose to initialise θ_r and θ_a at 1 which have the advantage of the simplicity and don't impose any particular hierarchy between rule and behaviour updates.

Computational studies performed with NetLogo [11] (for the multi-agent model) and Open Mole [12] (for the distributed processing) reveal several interesting stylized facts.

- All results presented in section 3 and 4 are robust under time scale endogeneization for $\nu \in [0.1, 0.9]$. Thus, asynchronous update and heterogeneity in time constants do not change metamimetic dynamics,
- The means of θ_r and θ_a per type converge toward values which are lower than 1 (around 0.5 for θ_r) (cf. fig. 5-A). Conformist agents are the population with the highest update frequency, they are the most concerned with what others think (fig. 5-A),
- The ratio $r_\theta = \frac{\theta_r}{\theta_a}$ is always lower than 1 (cf. fig. 5-A), indicating a clear hierarchy between action level time-scales and rule level time-scales. This is a behaviour that could be expected from rational agents (give your rule a chance before changing it) although but no such assumption has been done here.

[4]This is the case for the seminal model of [4] which results on emergence of cooperation are an artefact of the synchronous update of the agents behaviour.

[5] ξ will be typically a Gaussian noise of mean 0 with a variance of 1% to 5% of the measured quantity.

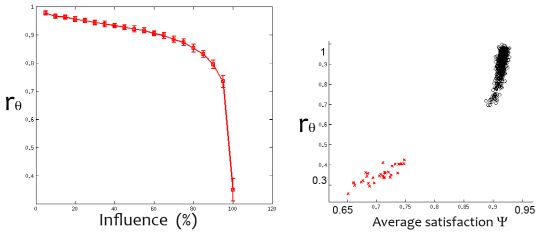

Figure 5: *A - left:* Effect of the influence ν on the ratio $r_\theta = \frac{\theta_r}{\theta_a}$ averaged value at the attractor. *B - right:* Scatter of r_θ against average satisfaction (red crosses correspond to $\nu = 1$ *i.e.* 100% of social influence).

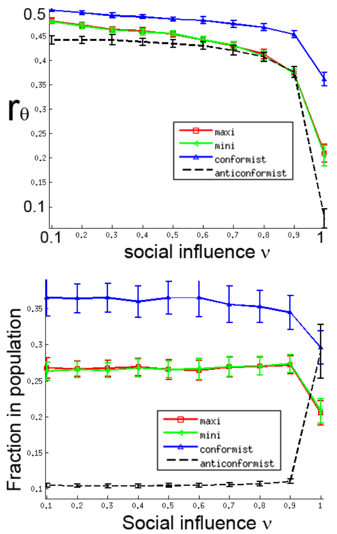

Figure 6: For $p = 0.4$, *A - Left:* Effect of the influence ν on averaged r_θ per type. *B - Right:* Effect of the influence ν on types distribution.

6 Conclusions

We tried to answer Smith's concern about the possibility to produce models where "types are an integral part of [their] predictive content" and are derived from "characteristics of the individual's memory-sensory system." Taking a spatial prisoner's dilemma as case study, we demonstrate that such models exist and propose a general methods for agents trait endogeneization in the framework of *metamimetic games*. This produces more constrained models with less parameters and consequently stronger predictions in the popperian sense of the term. Moreover, we proposed a state function which can qualify the evolution of an artificial social system. This function is related to the satisfaction of agents and could easily be interpreted as a component of the population welfare. Ψ reveals clear variations according to the parameters of the model and makes it possible to qualify different population states, although we deal with heterogeneous agents with non comparable utility functions. Our case study presented a minimal, thus incomplete, model, which nevertheless reveals several stylized facts:

- Cooperation is a robust phenomena,
- Population are heterogeneous with clear patterns with respect to agents type distribution
- Agents tends to prioritize the renewal of parts of their strategy in function of their proximity to action (behaviours are updated more frequently than rules of behaviour);
- The strength of a social dilemma changes types distribution in population and favours payoffs-based types,
- Payoffs-based types are the less satisfied agents and the population "well-being" measured in terms of averaged global satisfaction decreases as the strength of the dilemma increases.

All these stylized facts can be measured by studies in psychology, sociology or economy and are as many qualitatively measurable manifestations of the predictions on types distribution.

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